EPFL

7 – Quality Factor

ME-426 – Micro/Nanomechanical Devices

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Introduction

- Concept of Quality Factor
- Viscous damping
- Anchor losses
- Internal losses
- Damping in stressed beams

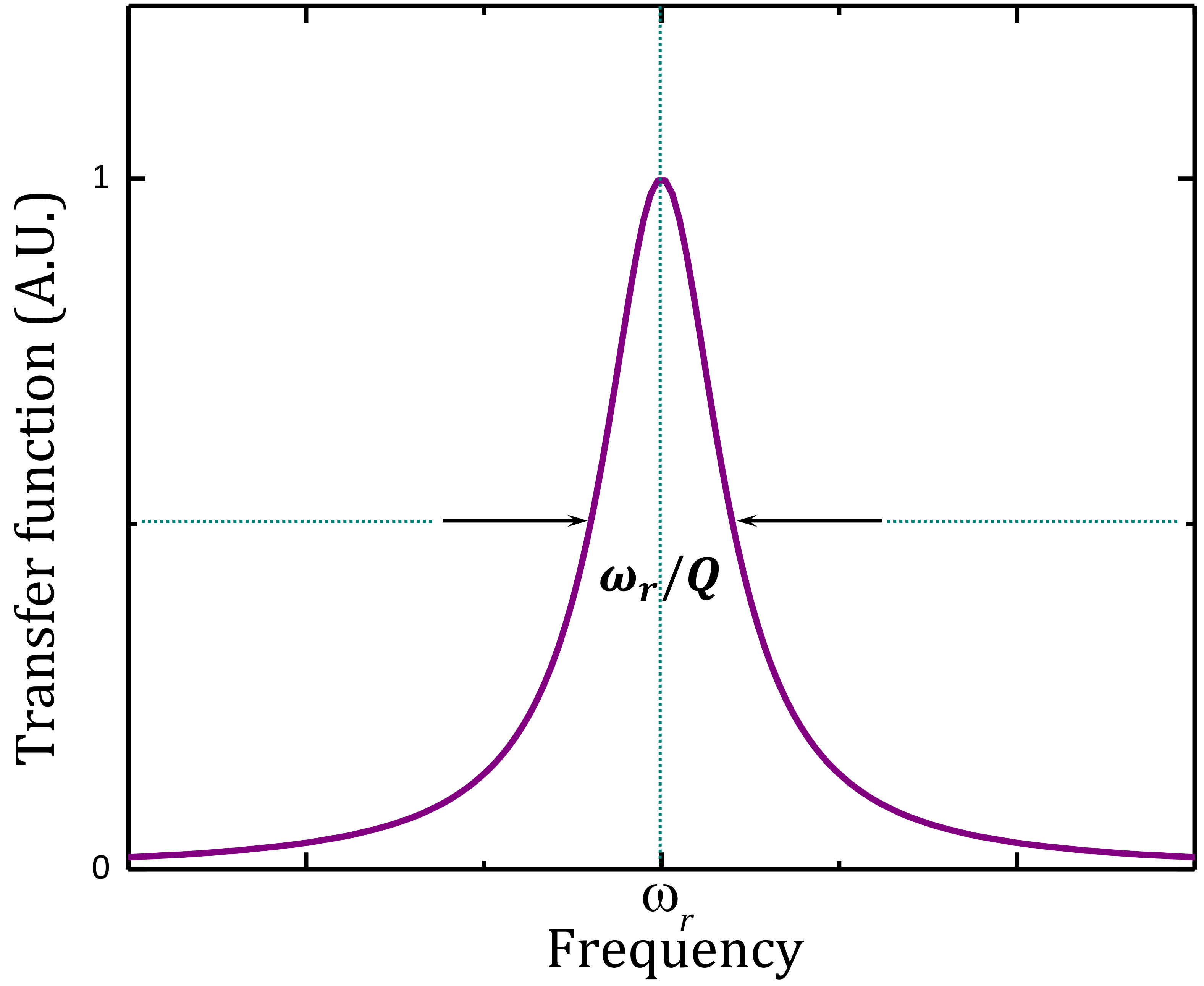
What is the Quality Factor?

What is a resonator?

- System which response to an external force is amplified at given frequencies

- Enhancement determined by the quality factor

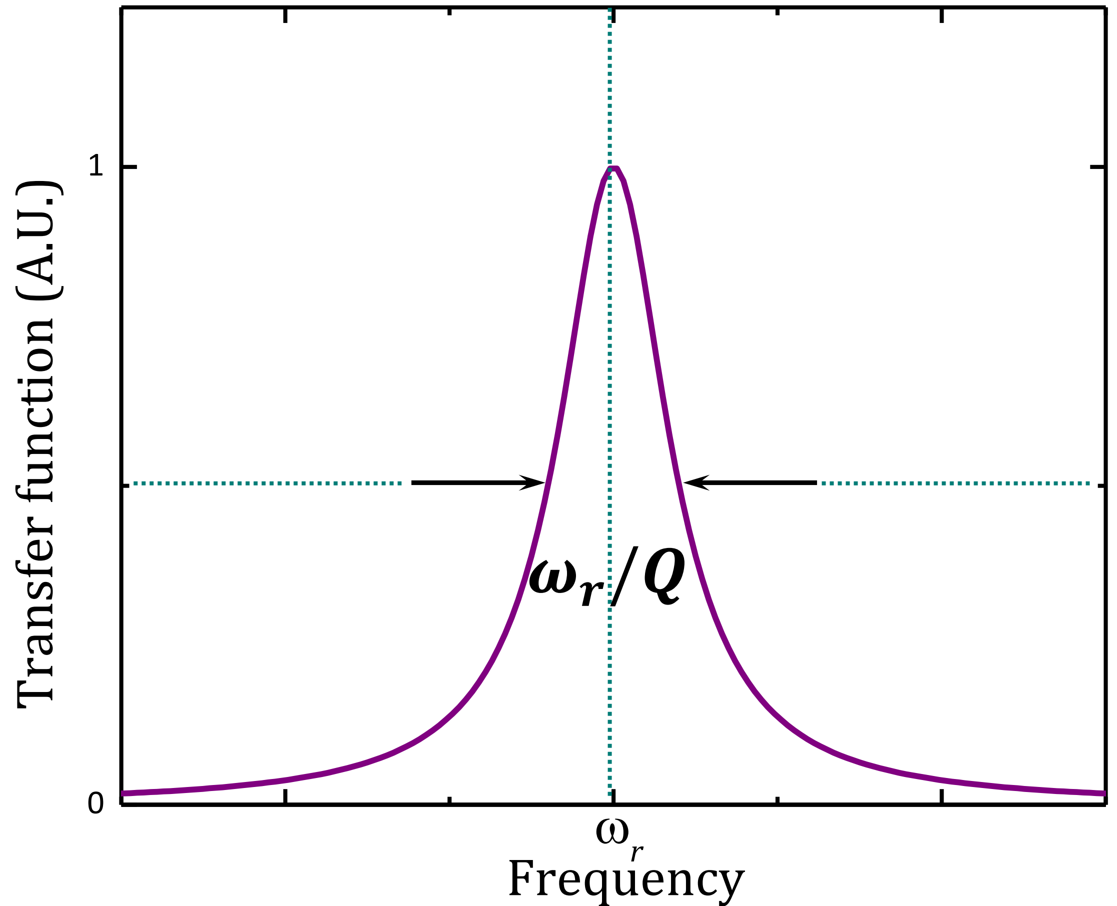
- But… what is the quality factor?
  - Measure of how good/bad a resonator is
  - Establishes level of interaction with the “outside”
  - $\boldsymbol{Q = 2\pi \frac{E_{stored}}{E_{lost}}}$
  - $\boldsymbol{Q = \frac{\omega_r}{FWHM}}$
  - $\boldsymbol{Q = \frac{S_{21}(\omega_r)}{S_{21}(\omega=0)}}$



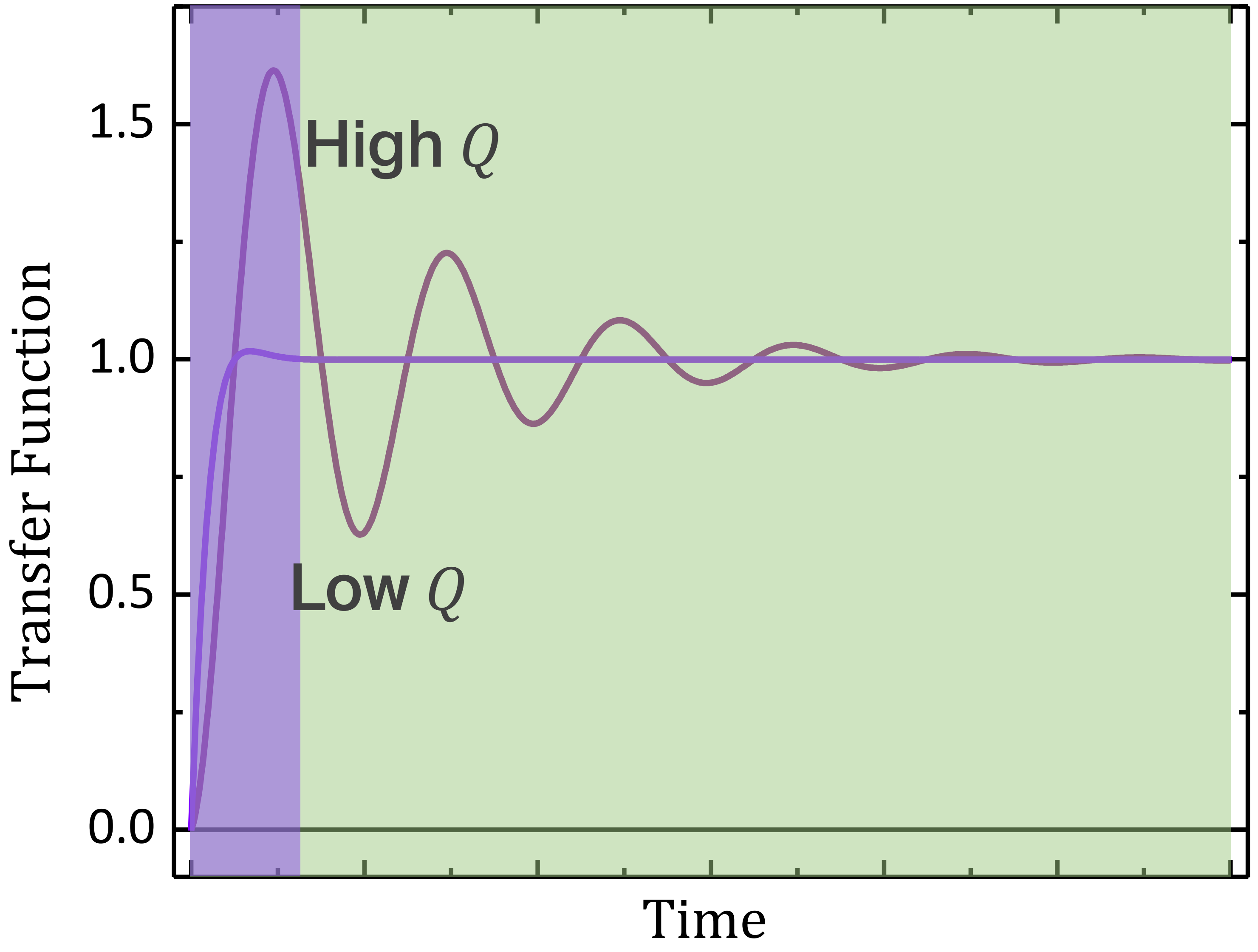
Is it better to have high or low quality factor?

- It depends on the application
- Trade-off
  - Less influence from external noise
  - Time required to settle down the transients $\tau \sim 2\pi \frac{Q}{\omega_r}$
    - Every mechanical structure has resonances
- If your application "resonates" – Q as high as possible

Resonator

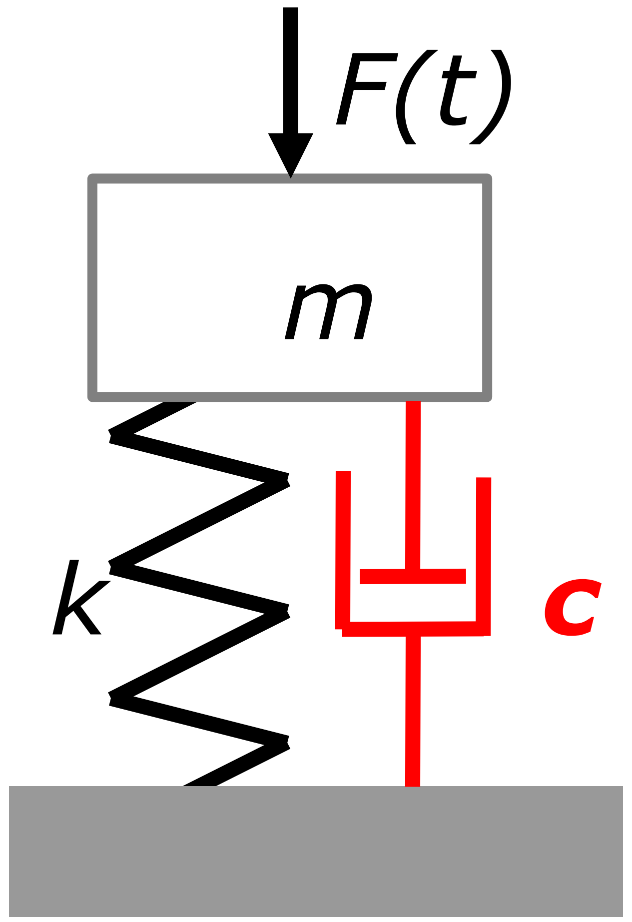


Response to a stepped stimulus



Damping

- Simple lumped model:



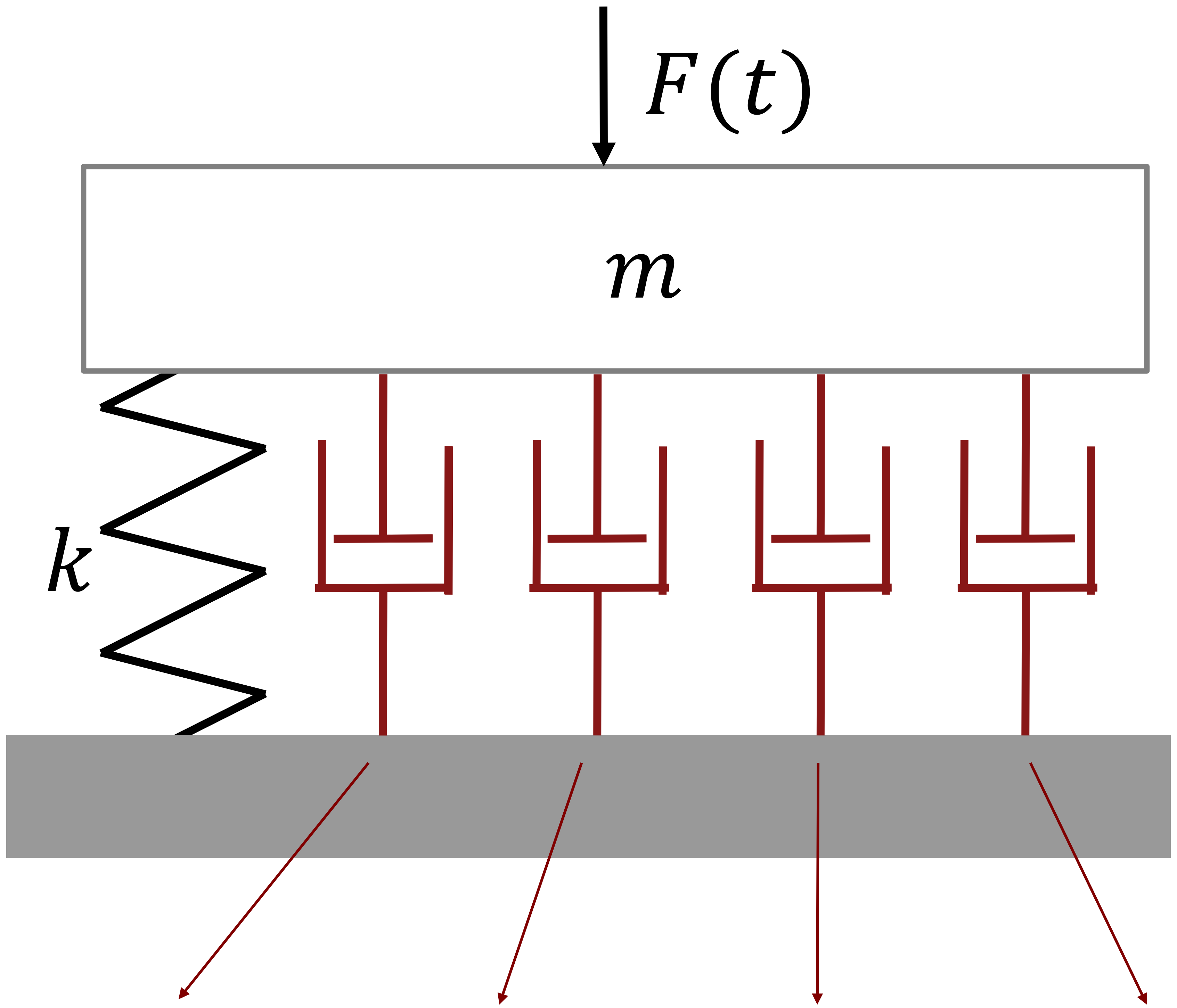
$$Q = 2\pi \frac{E_{stored}}{E_{lost}} = \frac{m_{eff}\omega_r}{c}$$

- But where is “c” coming from?

Possible damping sources in resonators

Damping

- A bit more extensive lumped model:



$$\boldsymbol{Q} = \boldsymbol{2\pi}\frac{\boldsymbol{E_{stored}}}{\boldsymbol{E_{lost}}} = \frac{m_{eff}\omega_r}{c}$$

$$\frac{1}{\boldsymbol{Q}} = \frac{1}{Q_{viscous}} + \frac{1}{Q_{internal}} + \frac{1}{Q_{clamping}} + \frac{1}{Q_{surface}} + \cdots$$

$$c = c_{viscous} + c_{internal} + c_{clamping} + c_{surface} + \cdots$$

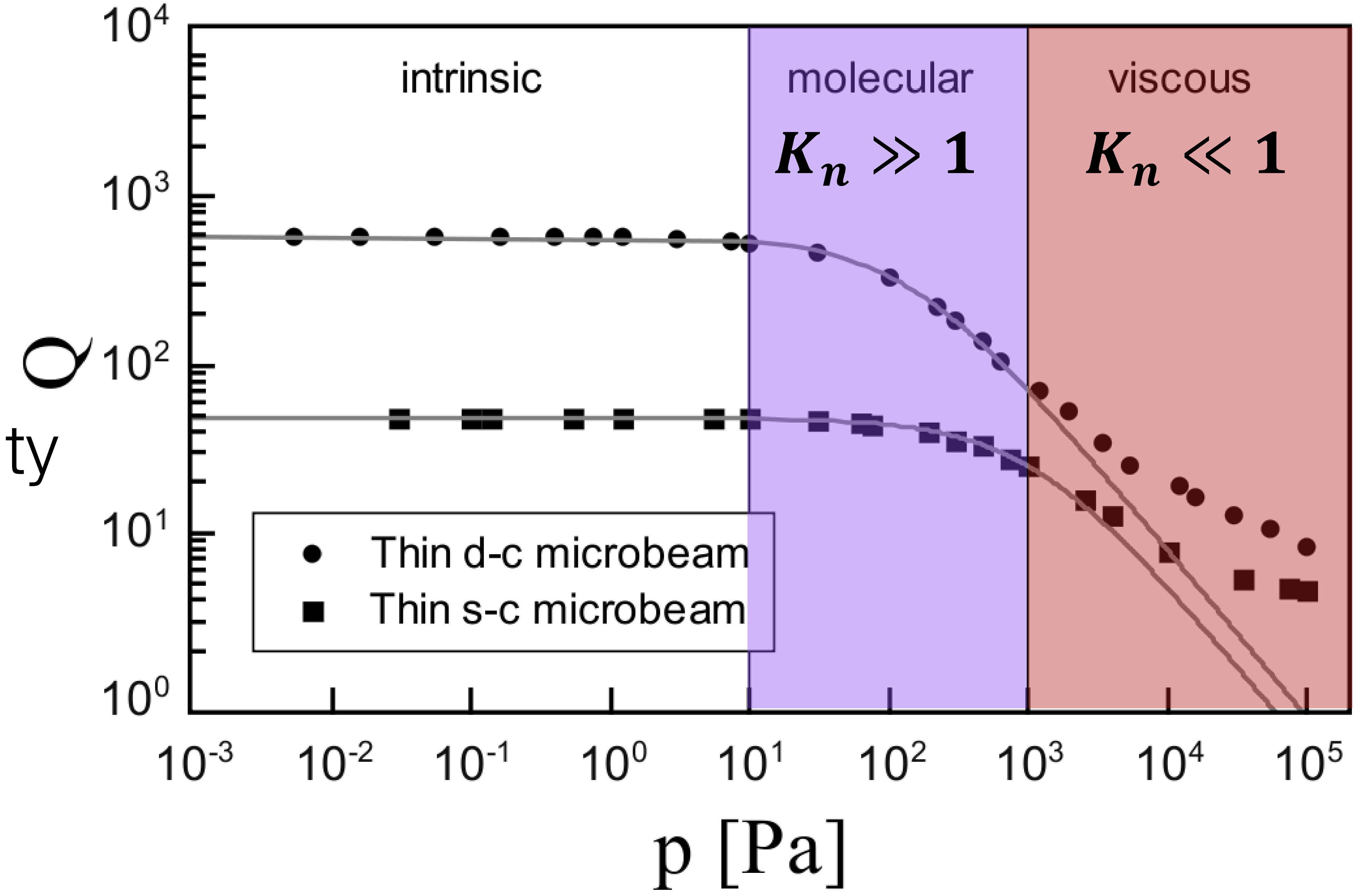
Viscous Damping

Viscous damping

- Caused by the interaction of the solid with the surrounding fluid

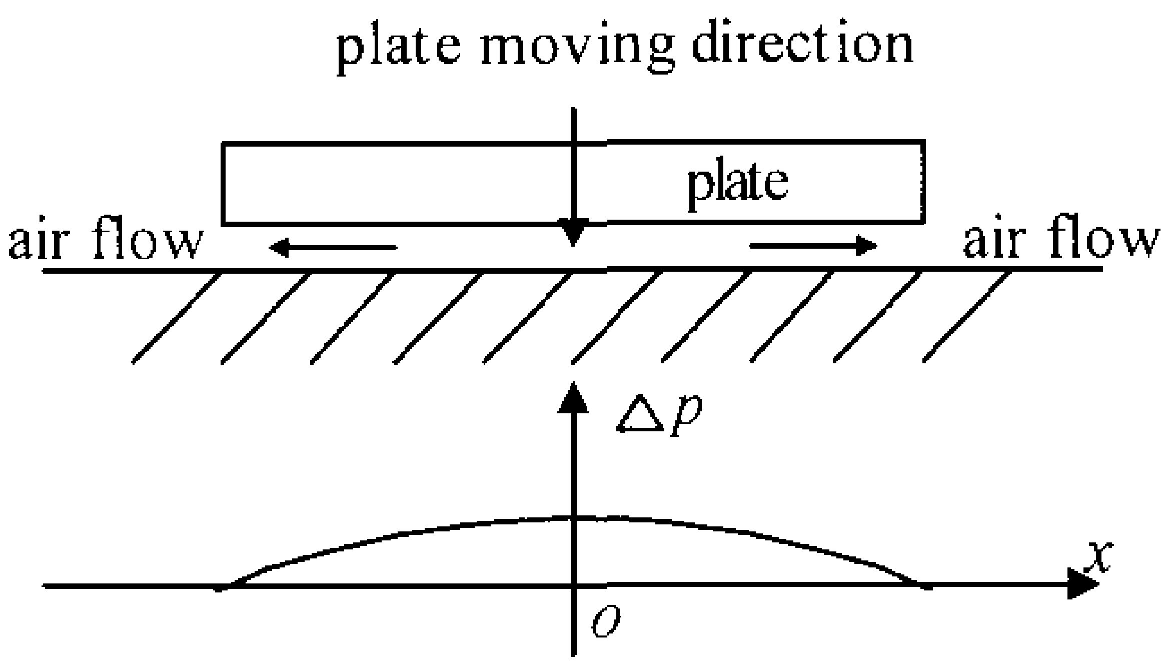
- Two regimes, depending on the Knudsen number
  - $K_n = \frac{\lambda_{mfp}}{L_c} = \frac{k_B T}{\sqrt{2}\pi d_{gas}^2 P L_c}$
  - Room temperature, P_{atm}: $\lambda_{mfp} \approx 70 \text{ nm}$

- Viscous regime is dominated by viscosity



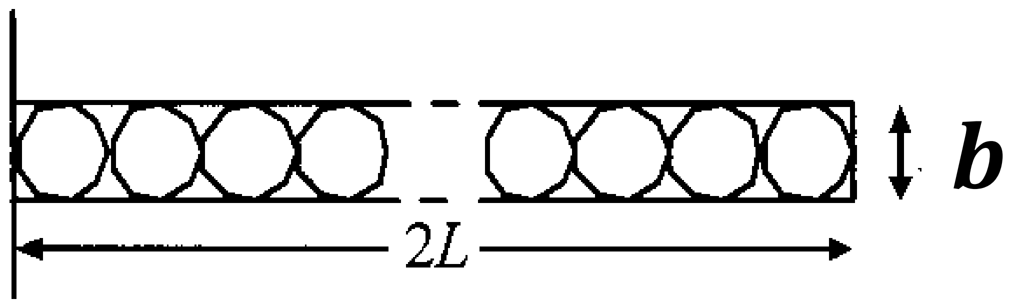
Two different types of interaction with the gas

- Squeeze film damping



$$Q_{sf,K_n<1} = \frac{\rho h f_r}{\mu b^2} g_0^3$$

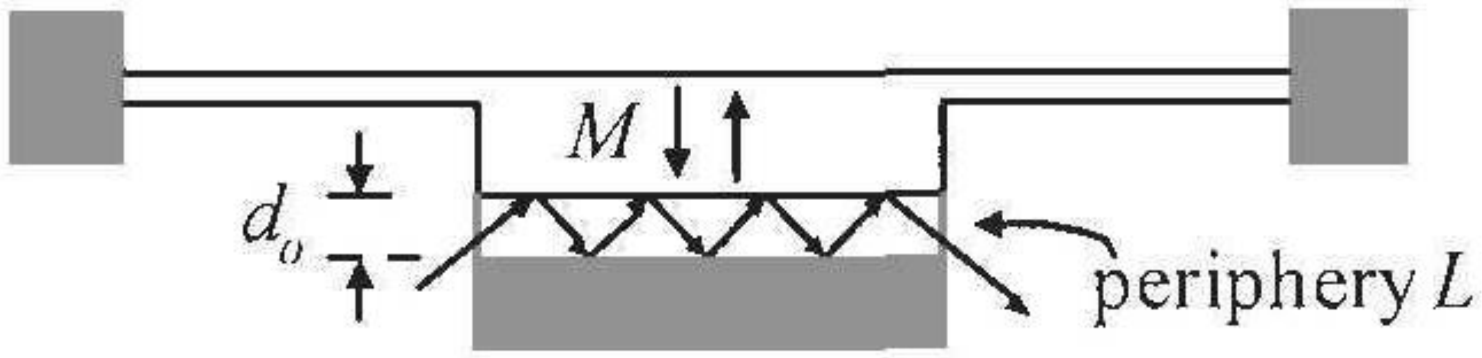
- Drag force damping



$$Q_{df,K_n<1} = \frac{\rho h f_r}{8\mu} b$$

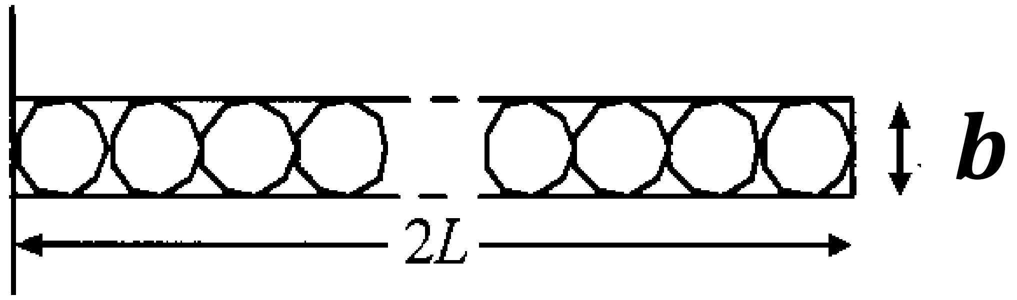
Two different types of interaction with the gas

- Squeeze film damping



$$Q_{sf,K_n>1} \sim \rho h f_r \frac{g_0}{L_p} \frac{1}{P} \sqrt{\frac{RT}{m_{gas}}}$$

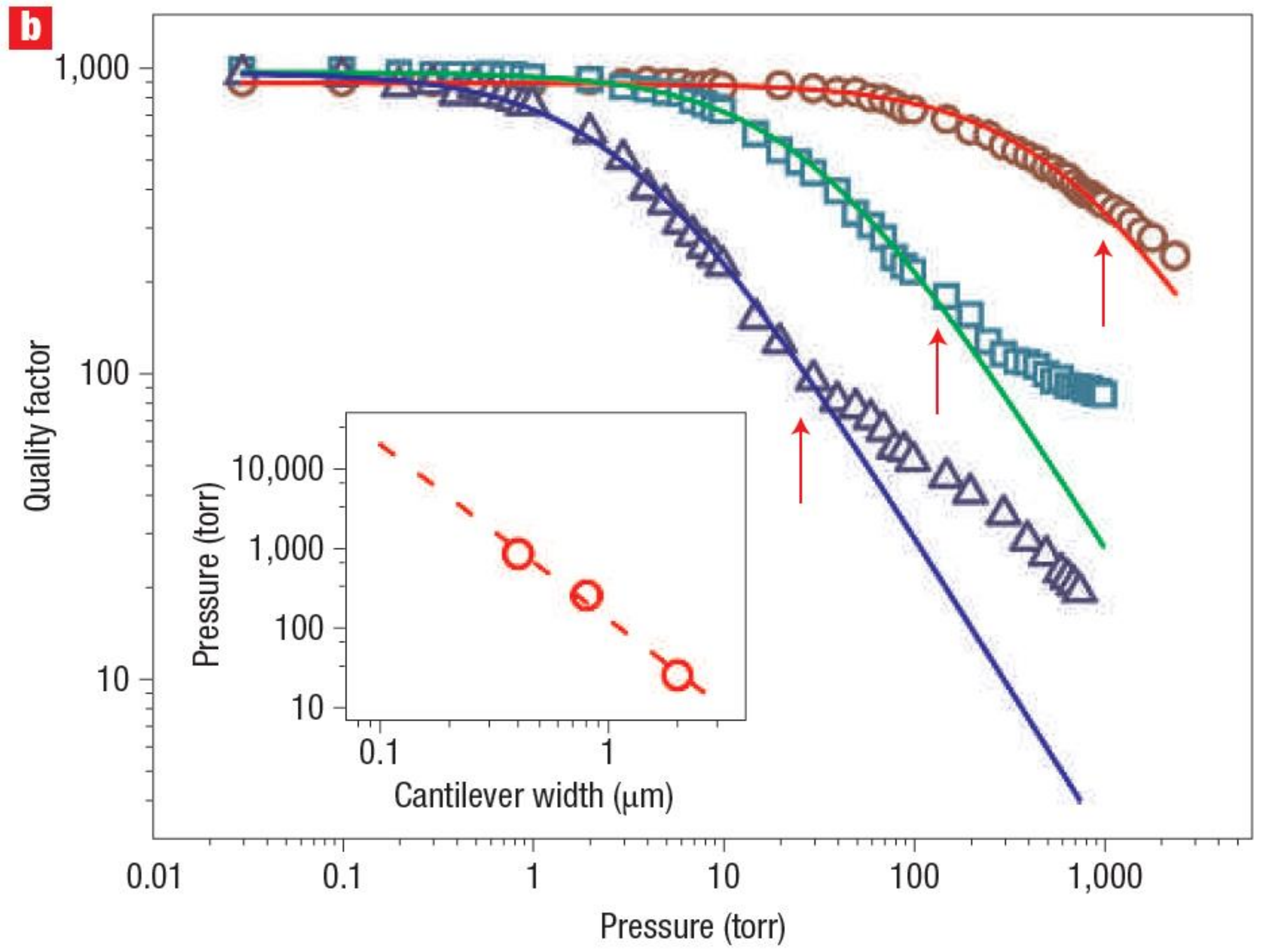
- Drag force damping



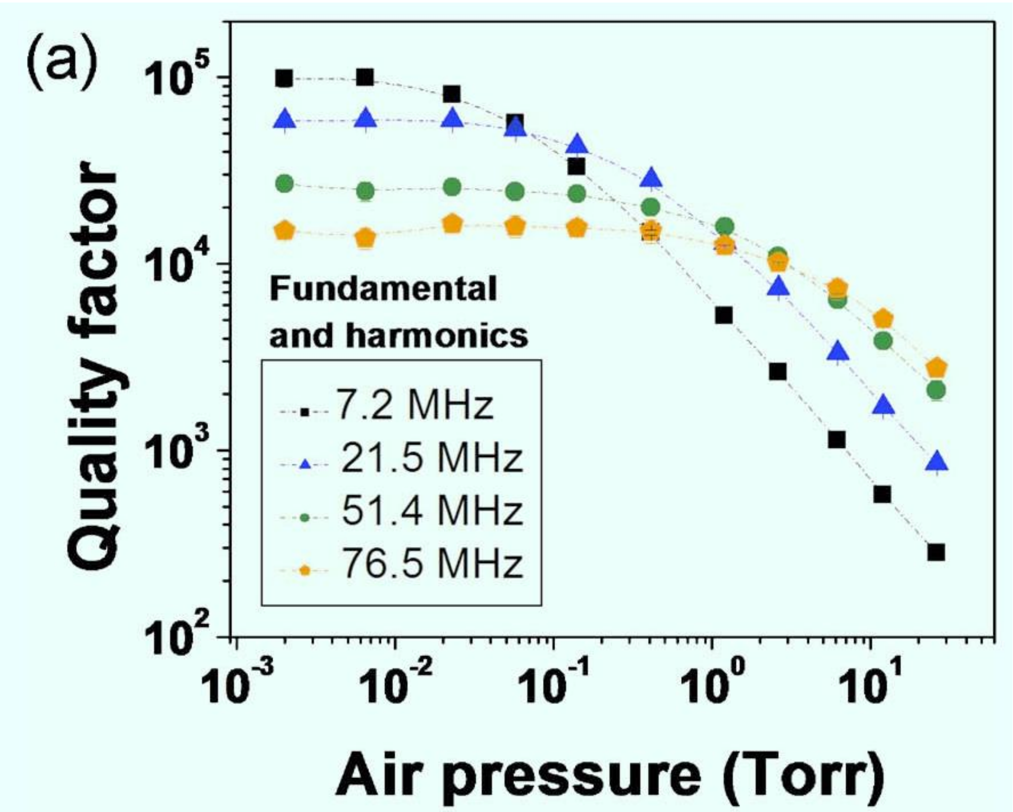
$$Q_{df,K_n>1} \sim \rho h f_r \frac{1}{P} \sqrt{\frac{RT}{m_{gas}}}$$

How do you optimize the design in order to minimize gas damping?

- Reducing dimensions, increases Knudsen number



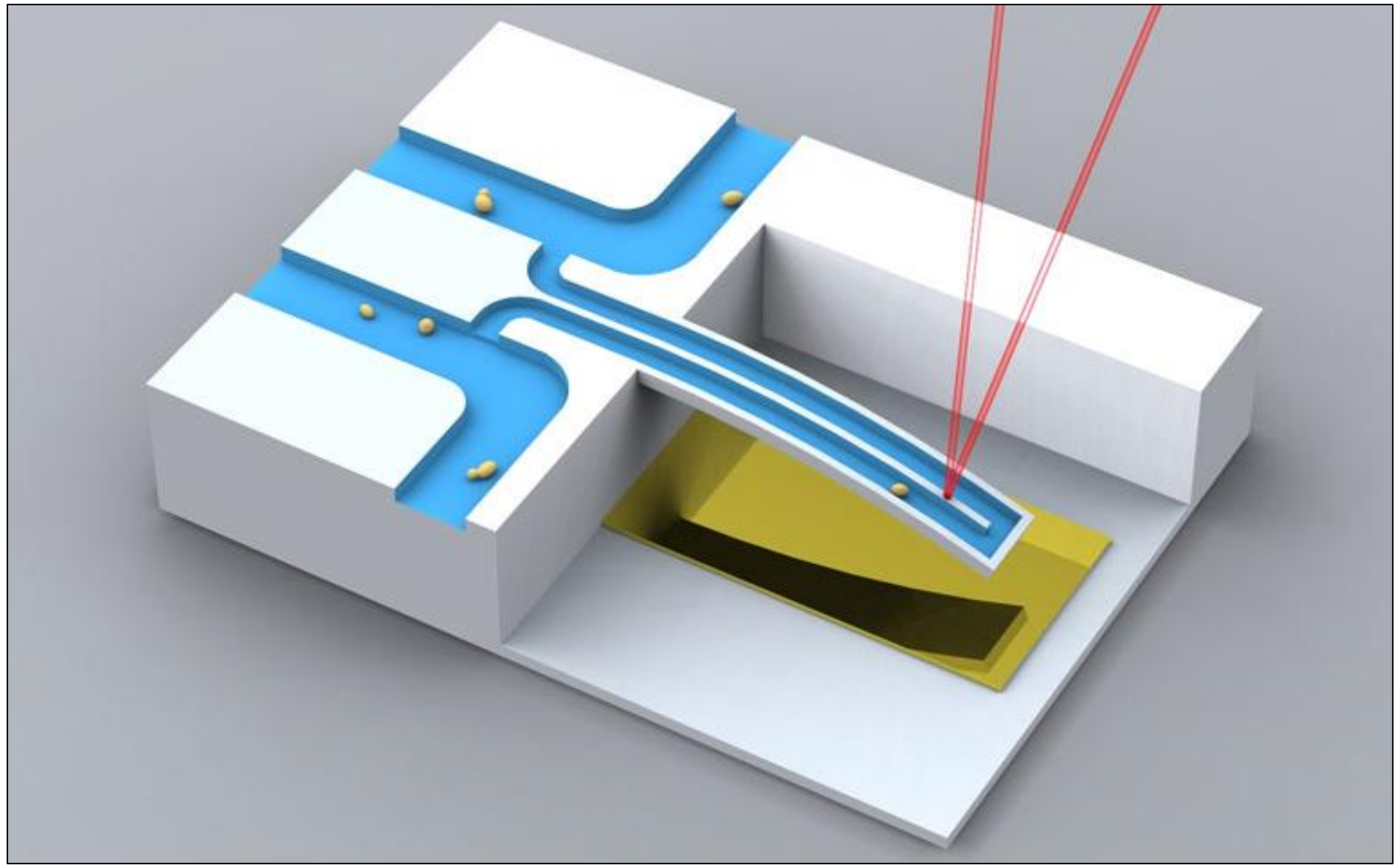
- Increasing mode number, increases frequency

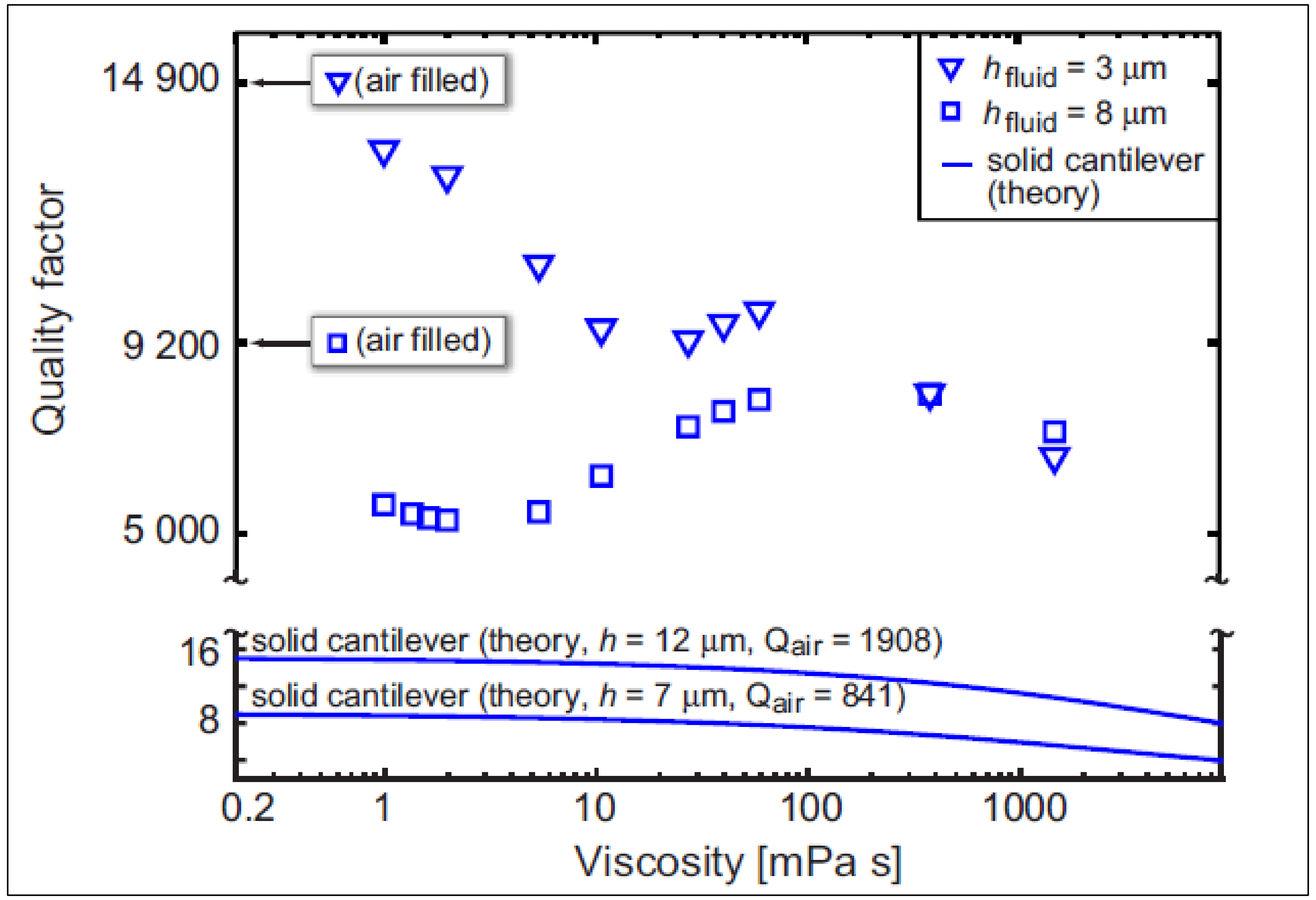


What if we are in liquid?

- Always in viscous regime
- Viscosity is so large that $Q < 20$ in most cases

- What if we put the liquid inside, instead of around the resonator?

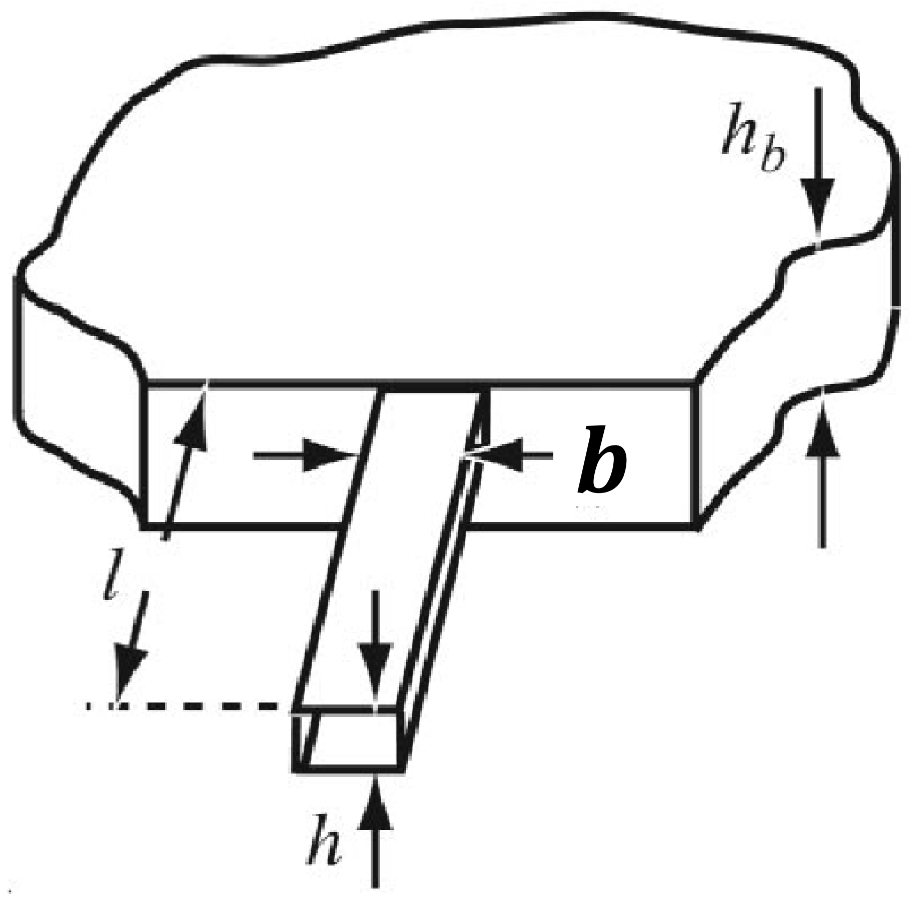




Clamping losses

Clamping losses

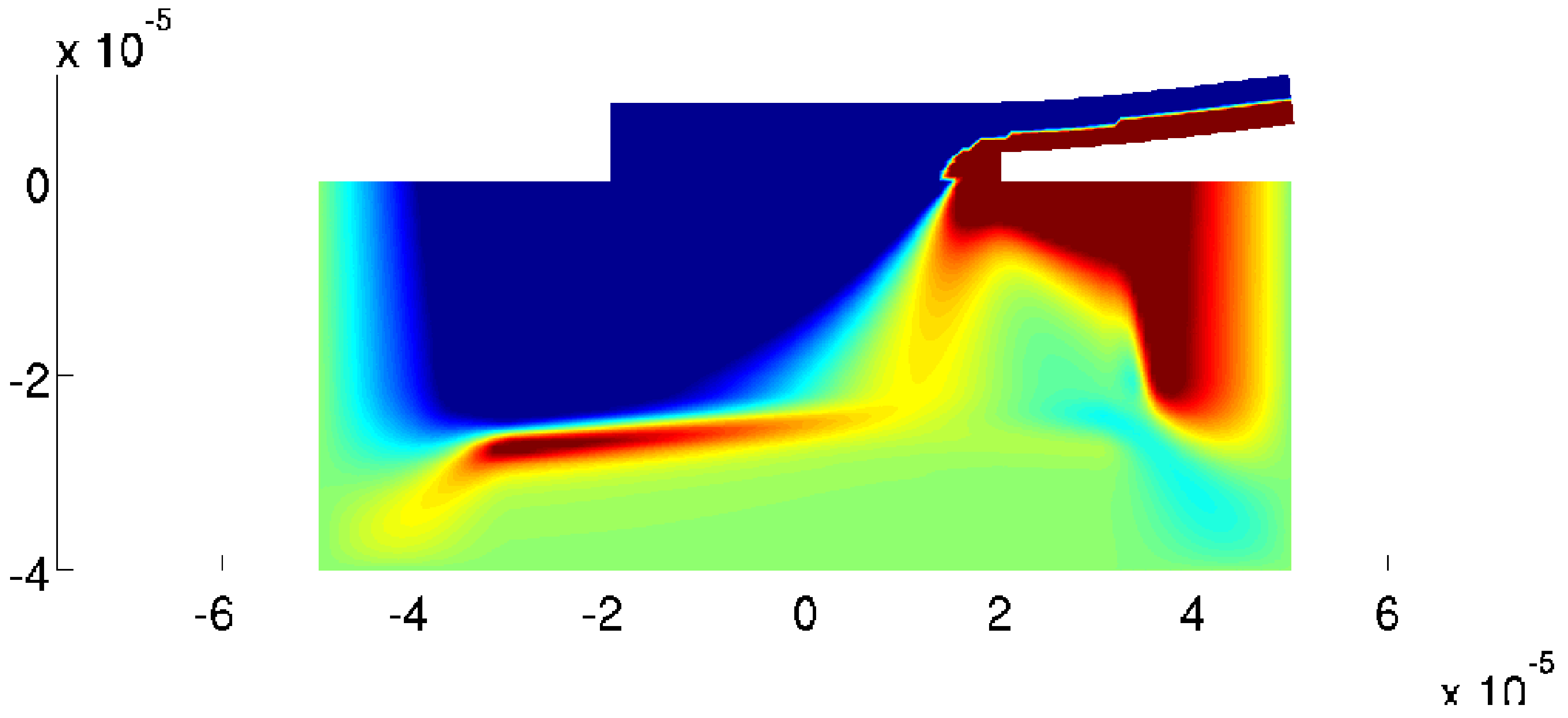
- In the boundary with the frame, the resonator sends acoustic energy away
  - This energy is lost
- This damping mechanism is difficult to model in general, since many different options for clamping exist
  - As a thumb rule, if $h_{substrate} \gg h_{device} \rightarrow Q_{clamp} \propto \frac{L}{b}$

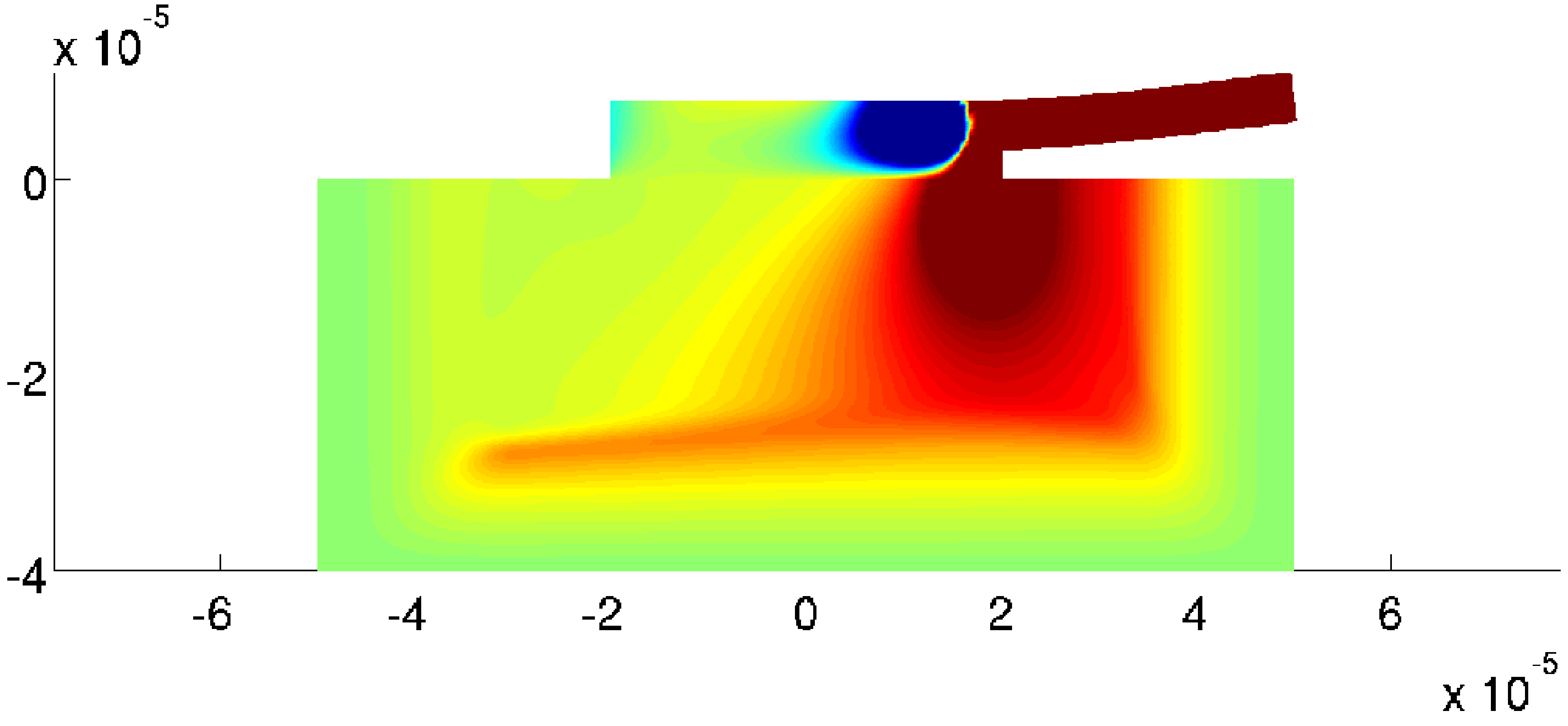


FEM with Perfectly Matched Layers (PMLs)

- Instead of perform analytical models, it is more effective to simulate our structure

- To do this, we need to input into the model Perfectly Matched Layers (PMLs) which absorb all energy that arrives to them

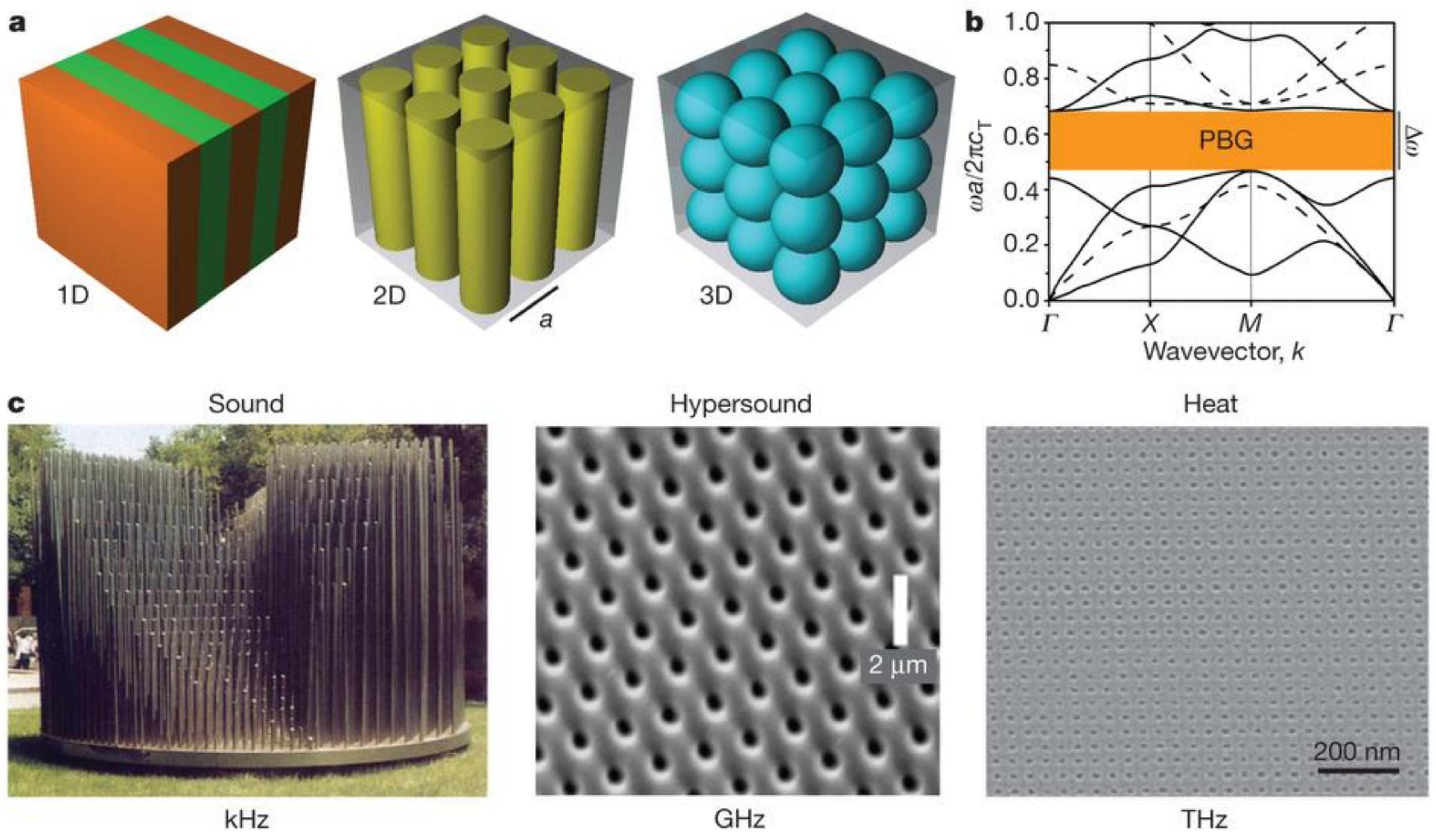




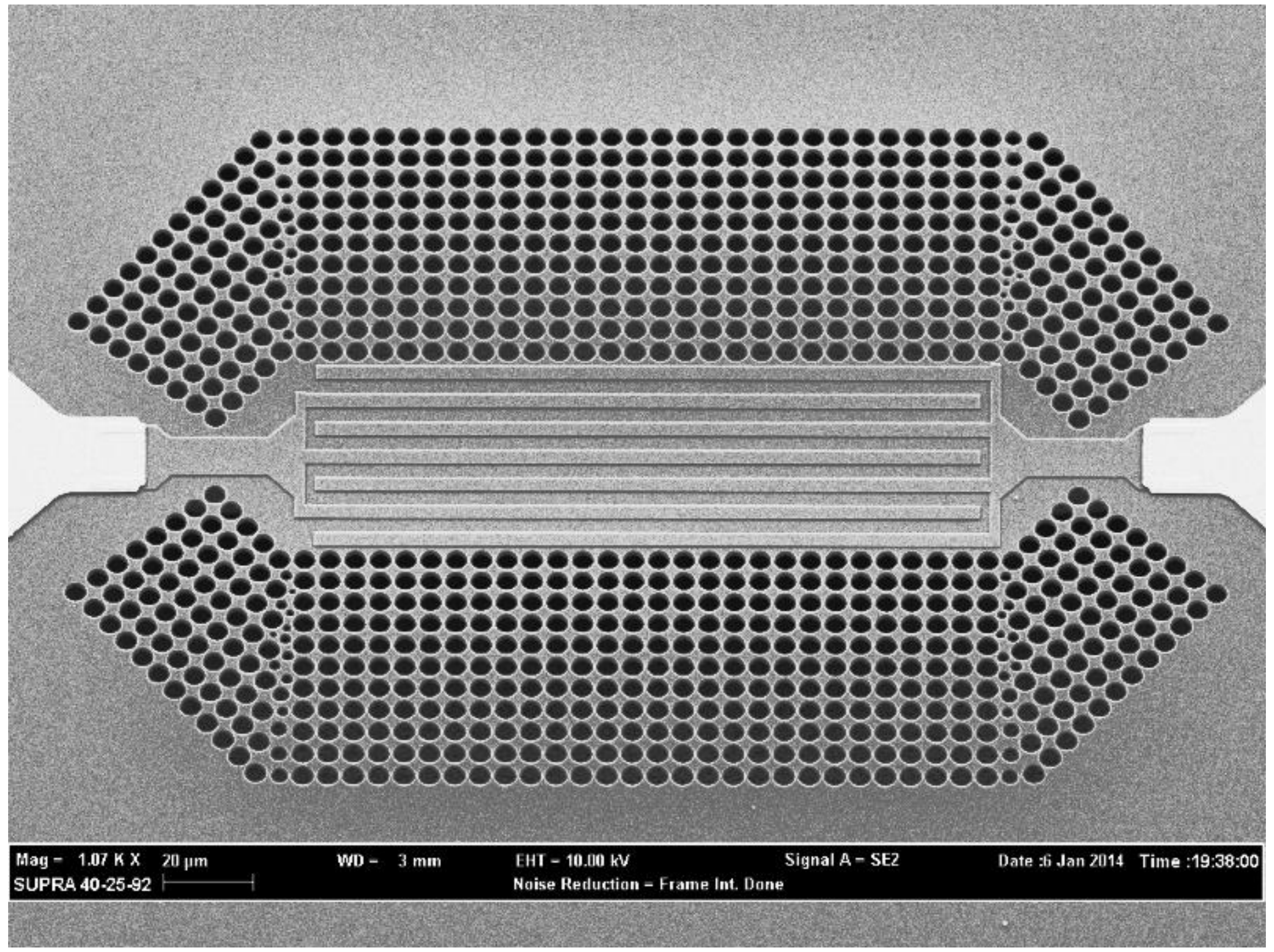
- This can be used to minimize clamping losses:
  - Via anchor design
  - Via phononic crystals

Phononic crystals

- Use of scattering & interference to create band gaps
  - Range of wavelengths within which waves cannot propagate
- Repetitive structure in artificially structured material
- Density and/or elastic constants change periodically
- Unit cell dimension comparable to (acoustic) wavelengths to stop



Moldovan, Nature 2013

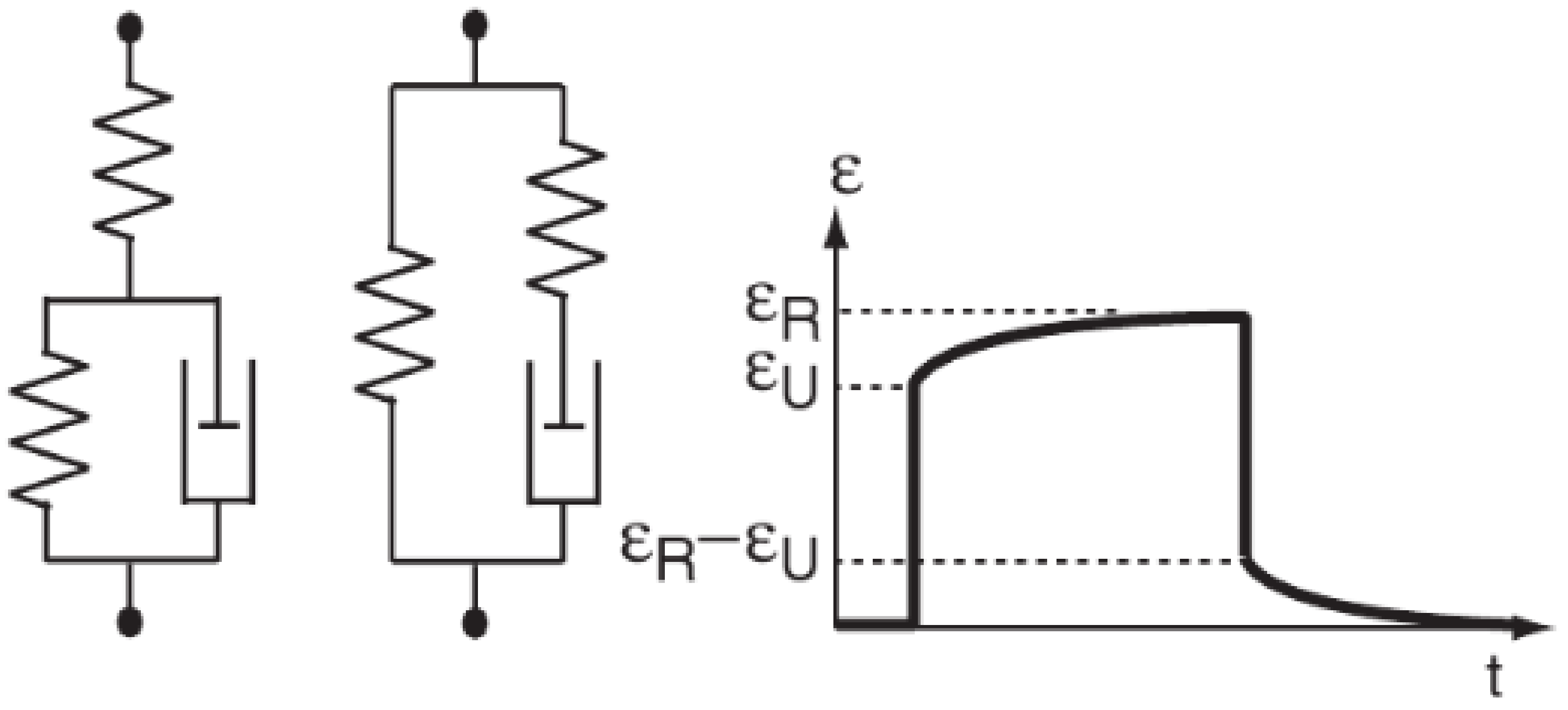


Wang, Proc. Solid-State Sens., Actuators 2014

Internal damping

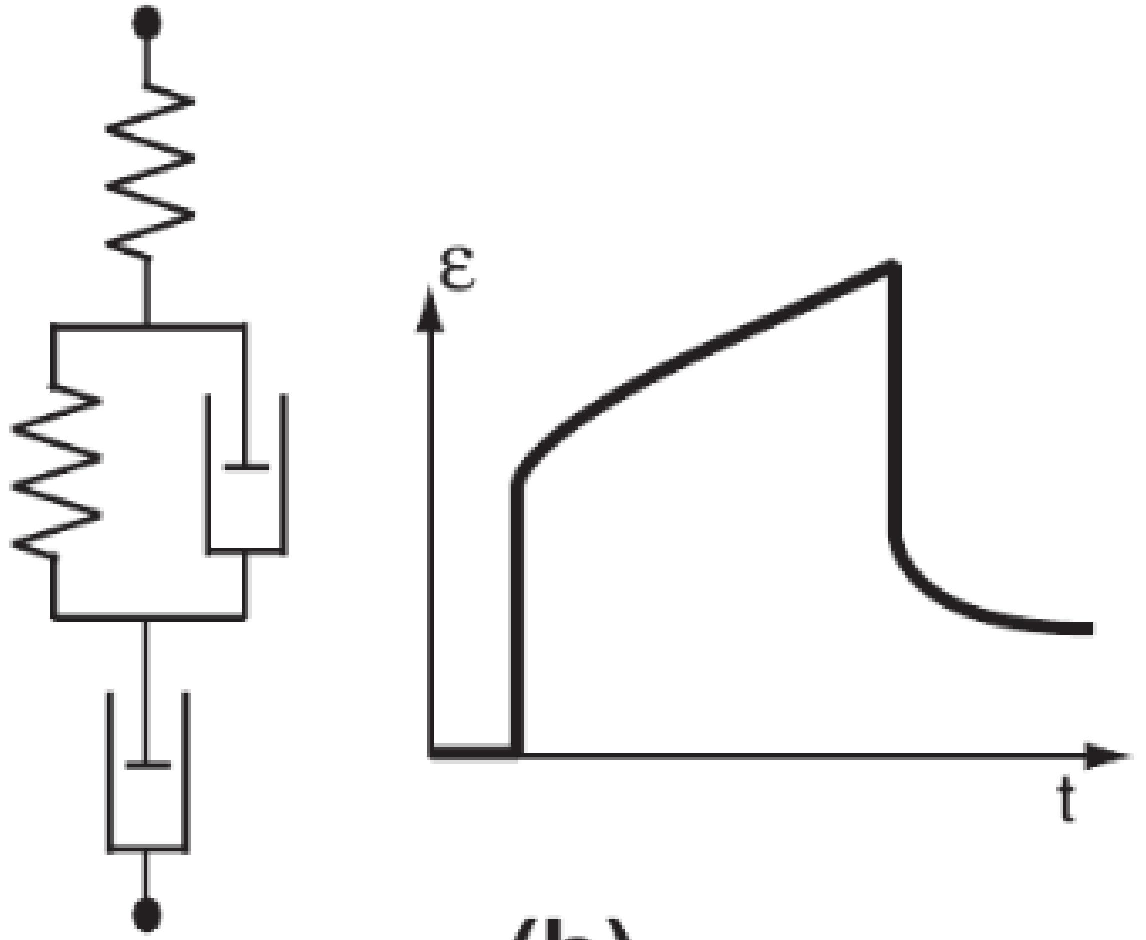
Internal material damping

- Anelastic solid (recoverable)



(a)

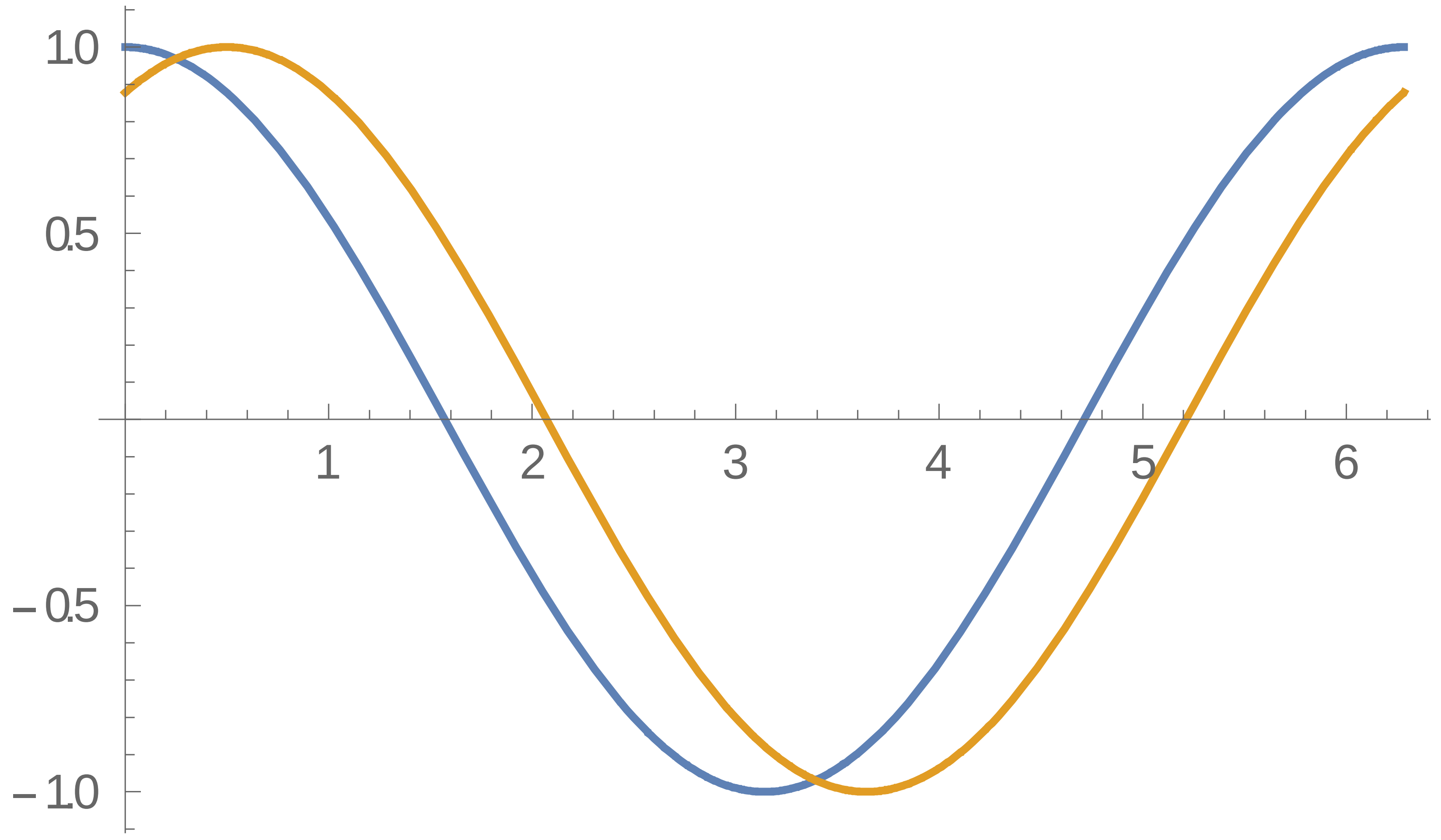
- Viscoelastic solid (non-recoverable)



(b)

Internal material damping

- Strain and stress do not go in-phase
  - $\varepsilon(t) = \varepsilon_M \cos(\omega t) = \Re\left(\varepsilon_M e^{j\omega t}\right)$
  - $\sigma(t) = \sigma_M \cos(\omega t + \delta) = \Re\left(\sigma_M e^{j(\omega t+\delta)}\right)$
  - $\frac{\hat{\sigma}(t)}{\hat{\varepsilon}(t)} = \frac{\sigma_M e^{j(\omega t+\delta)}}{\varepsilon_M e^{j\omega t}} = E' + jE''$
  - $E_{complex} = E' + iE''$
  - $\sigma(t) = \varepsilon_M E' \cos(\omega t) + \varepsilon_M E'' \sin(\omega t)$



- Energy lost per cycle:

$$\Delta u = \int_T \sigma \, d\varepsilon = \int_T \sigma \dot{\varepsilon} \, dt = \int_T \left(\omega \varepsilon_M^2 E' \cos(\omega t) \sin(\omega t) + \omega \varepsilon_M^2 E'' \sin^2(\omega t)\right) dt$$

$$\Delta u = \frac{\omega \varepsilon_M^2 E''}{2} \frac{2\pi}{\omega} = \pi \varepsilon_M^2 E''$$

Internal material damping

- Energy Stored

$$u_{max} = \int_{T/4} \sigma \, d\varepsilon = \frac{\omega \varepsilon_M^2}{2} \int_{T/4} \left(E' \sin(2\omega t) + \frac{E''}{2}(1 - \cos(2\omega t)) \right) dt$$

$$u_{max} \approx \frac{\omega \varepsilon_M^2 E'}{2} \frac{2}{2\omega} = \frac{\varepsilon_M^2}{2} E'$$

- Q

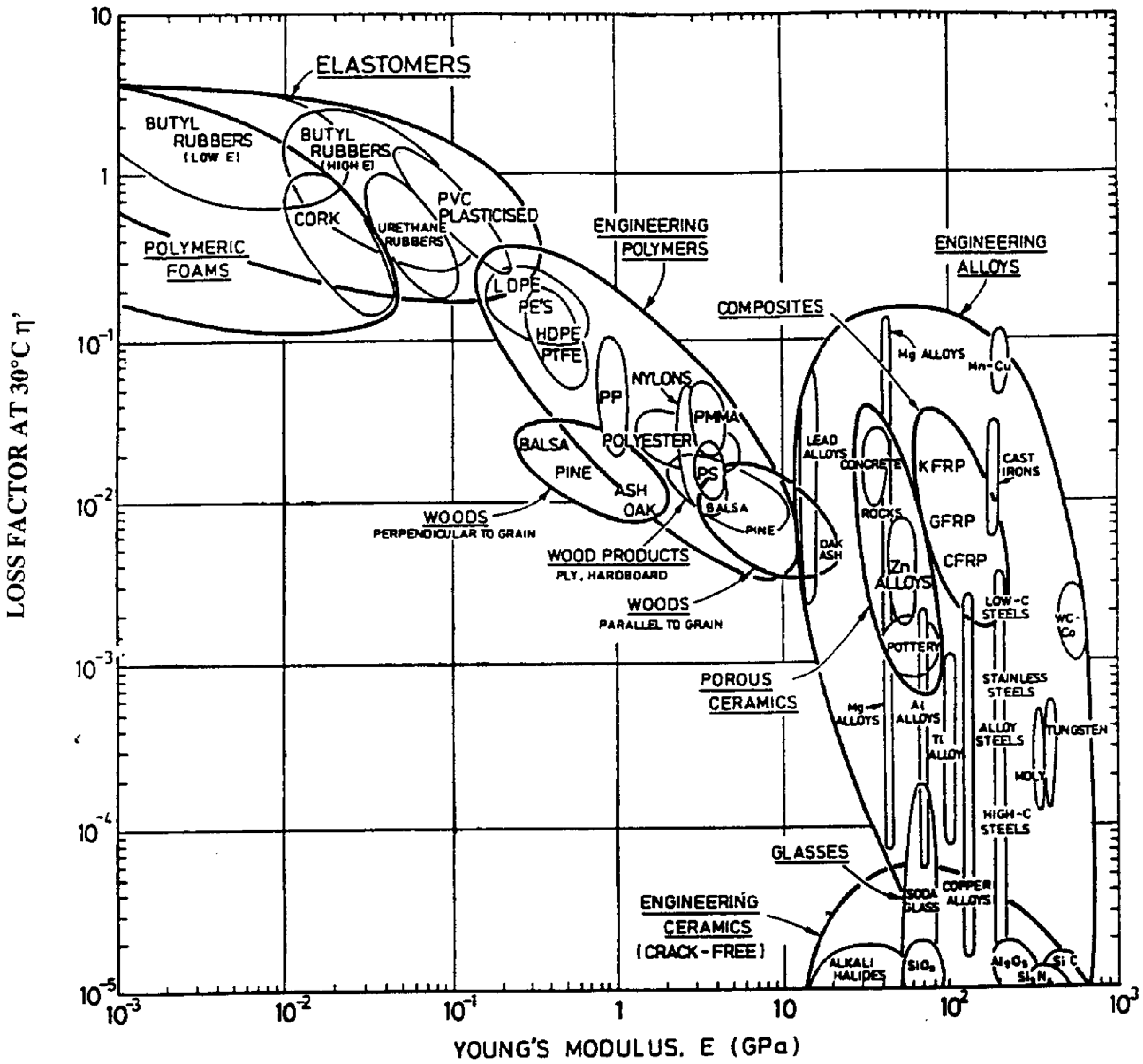
$$Q = 2\pi \frac{U_{max}}{\Delta U} = 2\pi \frac{\frac{\varepsilon_M^2}{2} E'}{\pi \varepsilon_M^2 E''} = \frac{E'}{E''} = \frac{1}{tan(\delta)}$$

Thermal activation

- The anelastic relaxation stems from thermally activated motion of various defects
- This motion can be described by Arrhenius' law $\propto \boldsymbol{e}^{\frac{E_{activation}}{k_B T}}$
  - This means that cooling down will "freeze" defects and thus improve Q

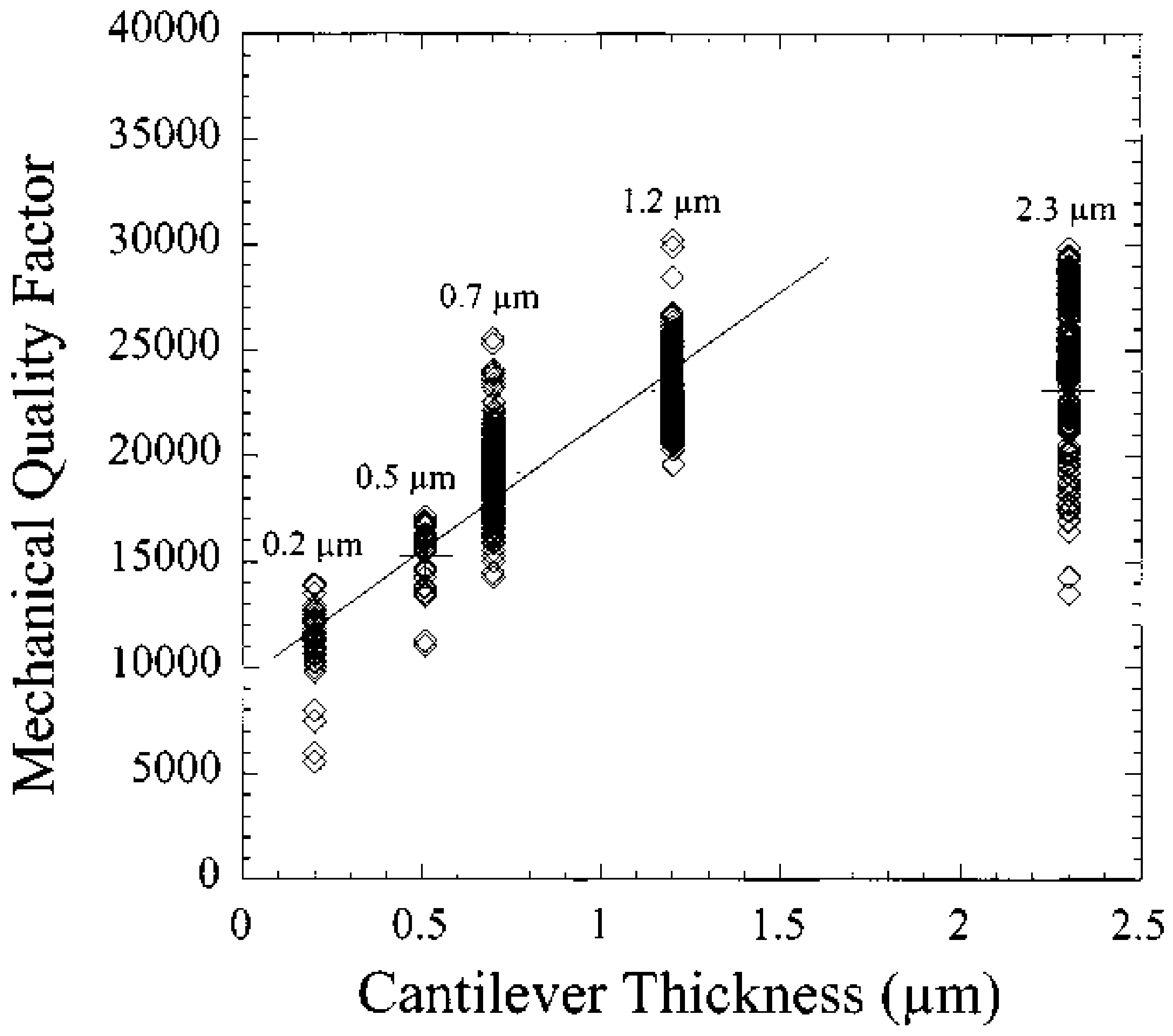
- Each material has a different "δ"
- Which one do you think is best?
  - Single crystalline
    - Si
    - Diamond
  - Poly-crystalline
  - Ceramics
  - Metals
  - Polymers

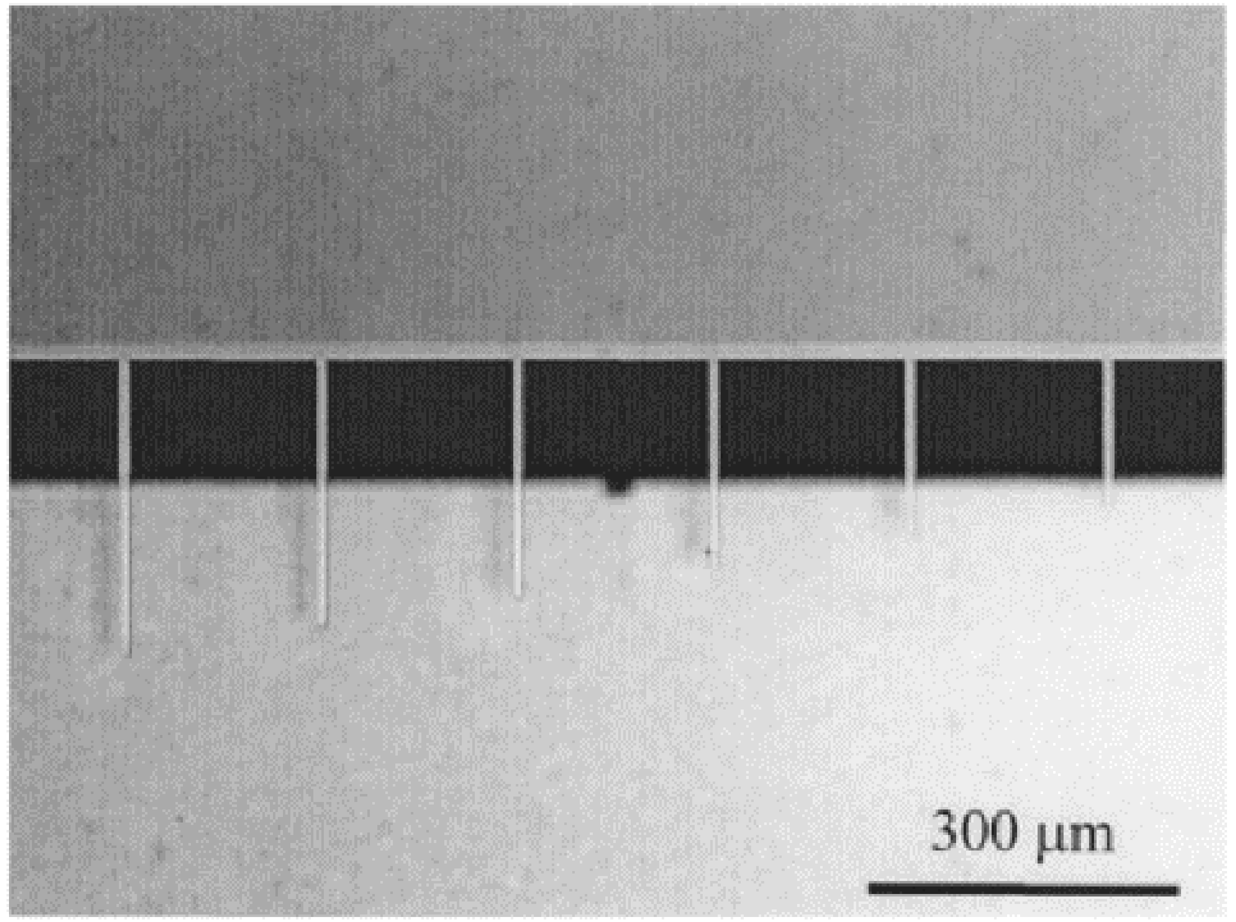
Material comparison

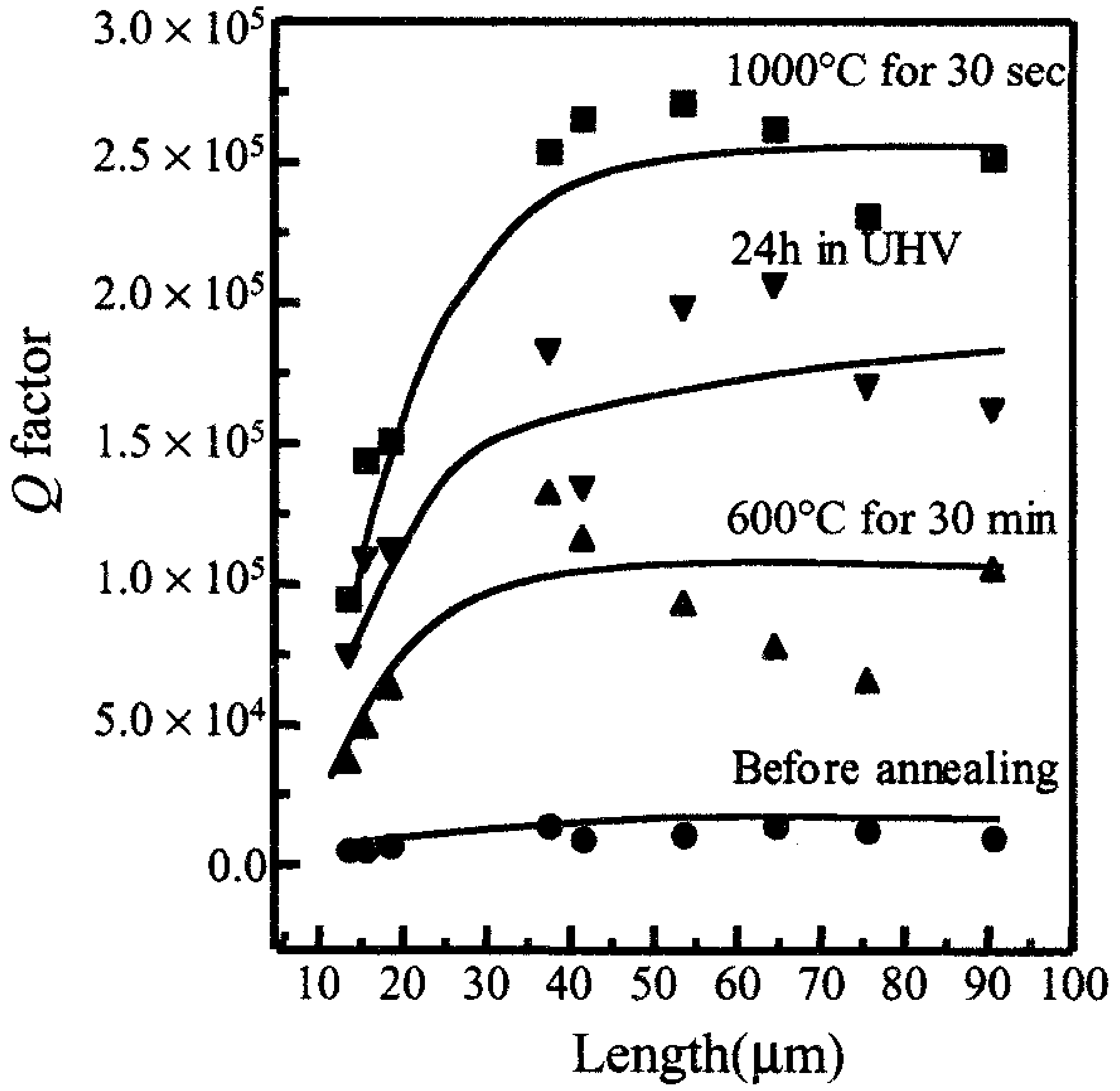


Surface effects

- Quality factor drops as a function of thickness
  - this is a clear consequence of a surface loss mechanism







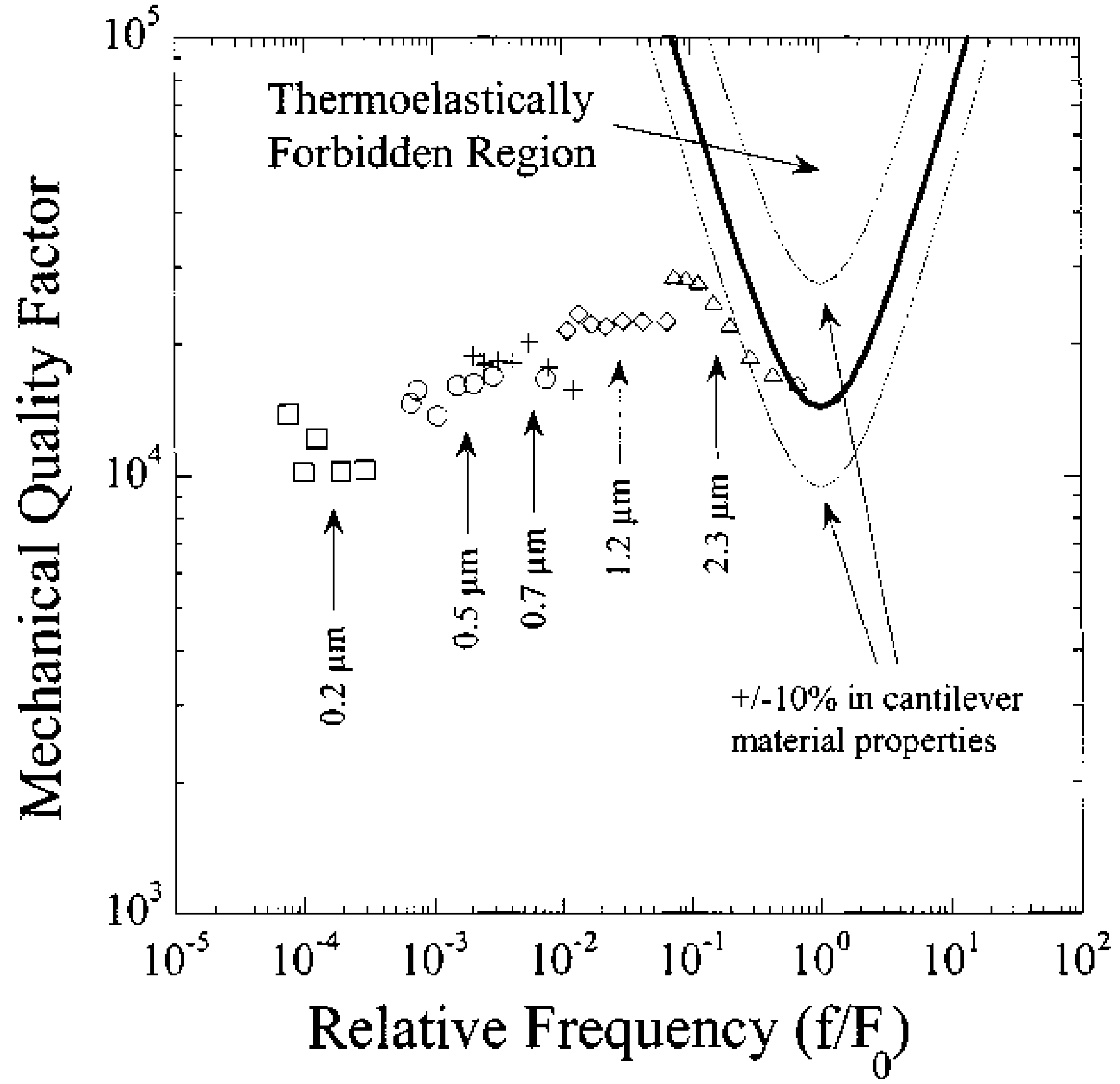
Thermoelastic damping

- This is a particular case of internal damping that happens in every material

- Expansion during movement will have an associated change in temperature

$$\varepsilon = \alpha \cdot \Delta T$$

  - ΔT is local and will create a temperature gradient inside the device
  - This temperature gradient causes a heat flow which implies energy being lost



Pre-stressed beams

What happens if we put Tension in the beam?

$$\varepsilon(t) = \varepsilon_0 + \varepsilon_M \cos(\omega t)$$

$$\sigma(t) = \sigma_0 + \sigma_M \cos(\omega t + \delta) = \sigma_0 + \varepsilon_M E' \cos(\omega t) + \varepsilon_M E'' \sin(\omega t)$$

- Energy lost per cycle:

$$\Delta u = \int_T \sigma \, d\varepsilon = \int_T \sigma \dot{\varepsilon} \, dt = \frac{\omega \varepsilon_M^2 E''}{2} \frac{2\pi}{\omega} = \pi \varepsilon_M^2 E''$$

- Energy stored:

$$u_{max} = \int_{T/4} \sigma \, d\varepsilon \approx \sigma_0 \varepsilon_M + \frac{\varepsilon_M^2}{2} E'$$

- Remember – this is energy density, per unit volume

What happens if we put Tension in the beam?

- We can use:

$$\varepsilon(x,t) = \varepsilon_0 + \frac{1}{2L}\int_0^L \left(w(t)\phi'(x)\right)^2 dx + (y - y_0)w(t)\phi''(x)$$

- Energy lost per cycle:

$$\Delta U = \int_V \Delta u \, dV = \pi E'' \int_V \varepsilon_M^2 \, dV \approx \pi I E'' w_M^2 \int_0^L \phi''^2(x) \, dx$$

- Energy stored:

$$U_{max} = \int_V u_{max} \, dV \approx \sigma_0 A w_M^2 \frac{1}{2}\int_0^L \phi'^2(x) \, dx + \frac{E'}{2} I w_M^2 \int_0^L \phi''^2(x) \, dx$$

- And then:

$$Q = 2\pi \frac{U_{max}}{\Delta U} = \frac{\sigma_0 A \int_0^L \phi'^2(x) \, dx + E' I \int_0^L \phi''^2(x) \, dx}{I E'' \int_0^L \phi''^2(x) \, dx} = \frac{E'}{E''} + \frac{\sigma_0}{E''}\frac{A}{I}\frac{\int_0^L \phi'^2(x) \, dx}{\int_0^L \phi''^2(x) \, dx}$$

Damping Dilution

$$Q = 2\pi \frac{U_{max}}{\Delta U} \approx \frac{E'}{E''} + \frac{\sigma_0}{E''}\frac{A}{I}\frac{\int_0^L \phi'^2(x)\,dx}{\int_0^L \phi''^2(x)\,dx} = Q_{\sigma=0}\left(1 + \frac{\sigma_0}{E'}\frac{A}{I}\frac{\int_0^L \phi'^2(x)\,dx}{\int_0^L \phi''^2(x)\,dx}\right)$$

$$Q \approx Q_{\sigma=0}\left(1 + \frac{\sigma_0}{E'}\frac{A}{I}L^2\frac{\int_0^1 \phi'^2(\xi)\,dx}{\int_0^1 \phi''^2(\xi)\,dx}\right)$$

- For Long, Thin, Stressed structures:

$$Q \approx Q_{\sigma=0}\frac{\sigma_0}{E'}\frac{A}{I}L^2\frac{\int_0^1 \phi'^2(\xi)\,dx}{\int_0^1 \phi''^2(\xi)\,dx} \rightarrow \text{Dilution factor}$$